Faculty of Science

Department of Mathematics
Seminar

Magnetic Geodesics and Generalization

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Abstract

Dynamical systems on 3-manifolds have been paid much attention along the time. In particular, magnetic trajectories are solutions of a second order differential equation (known as the Lorentz equation) and they generalise geodesics. A magnetic field on a Riemannian manifold is defined by a closed 2-form that helps, together with the metric, to define the Lorentz force. On the other hand, magnetic curves derive from the variational problem of the Landau-Hall functional, which is, in the absence of a magnetic field, nothing but the kinetic energy functional.

The dimension 3 is rather special, since it allows us to identify 2-forms with vector fields via the Hodge $\star$ operator and the volume form of the (oriented) manifold. Moreover, in dimension 3, one may define a cross product and therefore, the Lorentz equation may be written in an easier way.

The challenge is to solve the differential equation in order to find explicit solution, meaning the explicit parametrization for the magnetic trajectories. Nevertheless, this is not always possible and, because of that, it is necessary to understand the behaviour of the solution.

In 2014, together with J. Inoguchi, we defined the notion of magnetic map as a generalization of both magnetic curves and harmonic maps. A magnetic map is obtained as critical points of the LH functional, that is the energy functional together with a potential part.

A vector field can be thought of as a map from the manifold to its tangent bundle and since the tangent bundle carries a natural magnetic field obtained from its almost Kahlerian structure, we may ask when a vector field is a magnetic map?